Mathematical Modeling of Territory Formation in Parking Lots

Paul Kantlehner, Mentor: Dr. Allis

SYNOPSIS

There is a large gap in our understanding of how humans park territoriality in parking lots. There is research done on how we park in relation to points of interest, but little has been done to examine the individual: how do we park day to day? Do we tend to park in a single space we find most desirable? We have observed a parking lot for an extended period of time and have collected data on where specific cars park day to day. The data suggests there is territorial parking so we have developed a mathematical model to reflect that. We have created a probability matrix based on this data which can be used to find the likelihood of a car staying in the same space the next day, moving spaces, or not parking in the lot entirely. This matrix can then be multiplied in the Markov Cain fashion to find the probability of a car parking in a specific place.

BACKGROUND

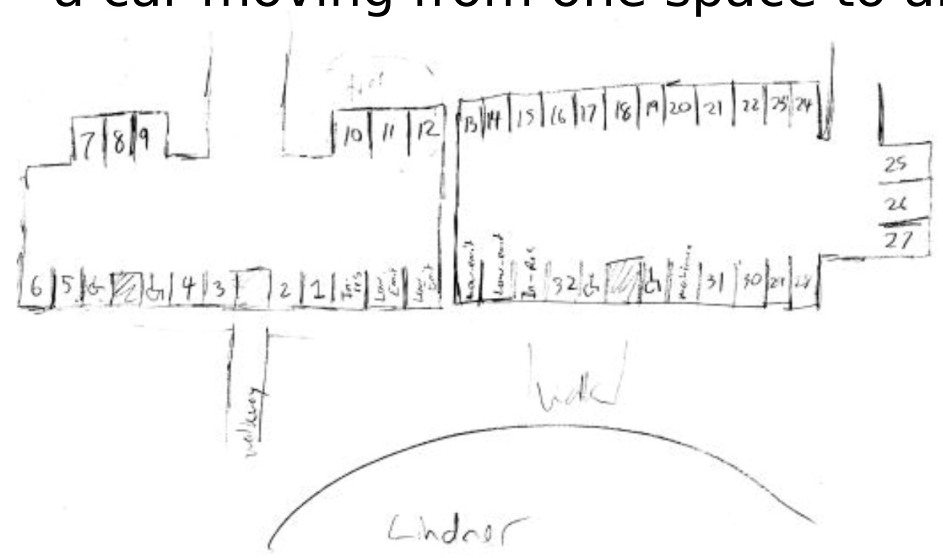
Mathematical modeling is the process of describing a system, usually in the real world, using mathematical concepts and language. This process varies from situation to situation but follows a basic routine of developing as simple model as possible then building up the model one piece at a time. This allows the process to be easily measured and back checked by comparison with the previous, simpler model to see how effective the addition was.

Our model is based around the concept of the random walk. The random walk is what we imagine a normal animal would do in a barren, featureless landscape: walking in a random direction before turning and walking in a different random direction over and over to explore. Taking this concept, it is easy to build a simulation where the animal moves then turns and moves again. Parameters are then added into the model to see how they match the actual data. If a parameter creates a model that is more accurate, the parameter is kept and the search for the next parameter begins. Once the movement of the animal is characterized, its territory is the area in which it travels most.

METHOD

To model territory formation in parking lots, we first made some assumptions before creating the model. The first assumption was that the cars, piloted by humans, would act like an independent animal and would only take up one space at a time. The second assumption we made was that the model would move in discrete steps: that is to say that all the cars would move at once. We did not look at the continuous flow of traffic in and out of the parking lot, we simply made the assumption that the cars came in and parked for the day and then all left at the end of the day.

These assumptions led us to began creating a model around Markov Chains. In our model, each discrete step would correspond to one day's parking arrangement of what cars parked where in the lot. Using the idea of Markov Chains, the transitions between each of these steps can be thought of as a square matrix where the column represents a parking spot and the row represents the possible spots that a car could move to. When a transition matrix is multiplied by a matrix of data, the result would give a prediction of the future state, what would be possible in the next step. These can be compounded to look further in the future. To build a model, we began with a 12 spot parking lot selected from behind the Linder Building at Elon University. To collect data, we took pictures of the parking lot at the same time every week day. We then mapped the car to the space location using the map below for each day. With that data, we could then create actual transition matrices. These are square matrices that contain the probability of a car moving from one space to another.



$$\begin{pmatrix}
a & 1-12a & 1-12a & 1-12a & 1-12a & 1-12a & 1-12a & 1-12a & 1-12a & 1-12a & 1-12a & 1-12a & 1-12a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a \\
\frac{1-a}{12} & a & a & a & a & a & a & a & a & a & a & a & a
\end{pmatrix}$$

To fit this data set, all of our models are square, 13x13 transition matrices where the first column and row stand for a car not parking in the lot and the rest representing one through twelve of the above labeled spaces. The simplest case of a model here would be a 13x13 matrix with every entry filled with the same probability, with no bias to leaving or going from a space. To build upon this, we first looked at the data we had collected for clues. The data hinted that there were two trends: one, that there was likely a higher probability of leaving the lot than moving from one space to another and, two, that cars did seem to park in the same spot. With the first observation in mind, we designed our first parameter to improve the model by isolating the lot leaving probability in the matrix shown above.

RESULTS

$$\begin{pmatrix}
0.076389 & 0.083332 & 0.083332 & 0.083332 & 0.083332 & 0.083332 & 0.083332 & 0.083332 & 0.083332 & 0.083332 & 0.083332 & 0.083332 & 0.083332 \\
0.0769676 & a+0.076389 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} \\
0.0769676 & 0.076389-\frac{a}{11} & a+0.076389 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} \\
0.0769676 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & a+0.076389 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} \\
0.0769676 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & a+0.076389 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} \\
0.0769676 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & a+0.076389 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} \\
0.0769676 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & a+0.076389 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} \\
0.0769676 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & a+0.076389 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} \\
0.0769676 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & a+0.076389 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} \\
0.0769676 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & a+0.076389 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} \\
0.0769676 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & a+0.076389 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} \\
0.0769676 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & a+0.076389 & 0.076389-\frac{a}{11} \\
0.0769676 & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & 0.076389-\frac{a}{11} & a+0.076389
\end{pmatrix}$$

Guided by the maximum likelihood function, our research led to the matrix above for the best model. After isolating the off parking lot probabilities, we kept their magnitudes and added to the model by introducing a parameter bias on the diagonal, as can be seen in the matrix. The diagonal corresponds to the probability of the car parking in the same spot the next day; if this model proved to be significant assuming the null hypothesis of the simplest case, it would indicate that territorial parking is indeed a strong possibility. When we found the maximum likelihood ratio using this model and the null hypothesis, we found the test statistic to have a significance of 24.1793. This is incredibly high and merits continuing to pursue this research from more solid and conclusive results.

FURTHER STUDIES

Our research has led to the development of a probability matrix which can be used to find the predict the likelihood of a car staying in the same space the next day, moving spaces, or not parking in the lot entirely. However, our model is rather simple, it does not take into account other factors such as spots that are more desirable than others, or specific areas where one is more likely to park. The model could be improved by adding more parameters. Our model also relies on the maximum likelihood function to provide validation for a better model. This was adapted from continuous territory formation and could be flawed or inaccurate in the discrete realm; it should be investigated further.

Another area of our model that requires future work is testing. This model has not been tested on any other lots. Thus, our model is limited by the size and people of this lot so more lots and more data would be needed to verify and develop the model. It would also benefit to test the model on a parking lot with cars that park in the same lot more regularly if possible. This could mean testing a lot with workers who operate on a more regular schedule than that of a college campus.